DIOPHANTINE EQUATION

WHAT IS DIOPHANTINE EQUATION?

Diophantine Equation is introduced by a mathematician Diaphantus of Alexandria. It is an algebraic equation having two or more unknown for which rational or integral solutions are required.

The first general solution of Linear Diophantine Equation gave by a mathematician Brahmagupta who used the complicated notations for solving such equations but Diophantus did not utilize complicated algebraic notation.

Linear Diophantine Equation of two variables is given by,

$$ax + by = c$$

Where, a, b and c are integers and x and y are variables. Few examples of Diophantine Equations are given below: -

$$ax^2 + by^2 = cz^2$$

$$x^2 + y^2 = 10z^2 \; x^2 + 1 = y^2$$

It is not necessary that all Diophantine equations are solvable, for example, $2x + 2y = 1$ there is no integer solution of this equation. Hence, the linear Diophantine equation have finite number of solutions, e.g. 3x = 6.

Hilbert proposed twenty-three most essential unsolved problems of 20th century and his tenth problem was the solvability a general Diophantine equation.

He also asked for a general method of solving all Diophantine equations. Furthermore, Glodbach's theory is his eighth problem. In 1930, Godel, Turing, Kleene develop the theory of computability and Universal Turing Machine invented by Turing in 1946 and discovered fundamental unsolvable problems.

However, in 1970, Yuri Matiyasevich proved that the Diophantine problem is not solvable by stating that "*there is no algorithm which, for a given arbitrary Diophantine equation, would tell whether the equation has a solution or not*".

Furthermore, Anderson and Ogilvy give a number of Diophantine equations with known and unknown solutions in 1988.

DIOPHANTINE ANALYSIS:

It is a process which required for finding solutions to Diophantine equations. Some examples of linear Diophantine equations along with their solution are:

EXAMPLE NO.1:

Find a particular and complete solution of a given equation

$50x + 60y = 30$

Solution:

$50x + 60y = 30$

First we find the Greatest Common Divisor (GCD),

$$60 = 1 \times 50 + 10$$
$$50 = 5 \times 10 + 10$$

Now apply the Extended Euclidian Algorithm,

$$10 = (1 \times 60) + 10(-1 \times 50)$$

Therefore, a particular solution is,

$$x_0 = -3$$
$$y_0 = 3$$

And the complete solution is $\begin{array}{l} x = -3 + 6n \\ y = 3 - 5n \end{array}$

EXAMPLE NO.2:

Find a particular and complete solution of a given equation

$25x + 35y = 20$

Solution:

$25x + 35y = 20$

First, we find the Greatest Common Divisor (GCD),

$$35 = 1 \times 25 + 10$$
$$25 = 2 \times 10 + 5$$
$$10 = 2 \times 5 + 0$$

Now apply the Extended Euclidian Algorithm, $\begin{aligned} 5 &= (1 \times 25) + (-2 \times 10) \\ &= (-2 \times 35) + (3 \times 25) \end{aligned}$

Therefore, a particular solution is,

$x_0 = 12$
$y_0 = -8$

And the complete solution is $\begin{aligned} x &= 12 + 7n \\ y &= -8 - 5n \end{aligned}$

EXAMPLE NO.3:

Find a particular and complete solution of a given equation

$2x + 6y = 5$

Solution:

$2x + 6y = 5$

First we find the Greatest Common Divisor (GCD),

$6 = 3 \times 2 + 0$

Therefore,

$GCD(2, 6) = 2$ Hence, there is no solution because 2 does not divide 5.